End Semester Examination

Electrodynamics
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April 28$^{\text{th}}$, 2025 (Morning Session)
Duration: 180 minutes.
Total points: 75

Please give arguments where necessary. If it is unclear from your answer why a particular step is being taken, full credit will not be awarded. Grades will be awarded not only based on what final answer you get, but also on the intermediate steps.

1. A sphere of radius R_1 has a uniform charge density ρ, except for a small hollow spherical region of radius R_2. Of course, $R_2 < R_1$. The center of the hollow region lies a vector $\vec{a}$ away from the center of the larger sphere.
   (a) Show that the electric field throughout the hollow region is uniform and find the value of this uniform electric field.
   (b) Find the electric potential at the center of the hollow region.

   7 + 8 = 15 points

2. A uniform steady current I flows up (in the direction $\hat{e}_z$) along a filamentary wire from spatial infinity. As it reaches the $z = 0$ plane, it spreads out radially outward on the plane maintaining azimuthal symmetry.
   (a) Find the magnetic field everywhere.
   (b) Show that the magnetic field satisfies all boundary conditions.

   9 + 6 = 15 points

3. A parallel plate capacitor (of plate areas A and plate separation d) has its volume between the two plates filled with two different simple dielectric materials of dielectric constants ε_1 and ε_2 and thicknesses d_1 and d_2 respectively. The plate next to material 2 is now given a potential $V > 0$, while the other plate is grounded. In all calculations, you may neglect edge effects. The materials also has Ohmic conductivities σ_1 and σ_2.
   (a) If the materials sustain the same current densities, find the electric fields in each materials.
   (b) Find the total and free charge densities on the interface between the two dielectrics.

8 + (4 + 3) = 15 points

4. A simple dielectric sphere of radius R and dielectric constant ε is placed in a uniform electric field $\vec{E}_0$.

   (a) Find the electric field everywhere.

   (b) Show the electric field you obtained in the previous part satisfies all boundary conditions.

9 + 6 = 15 points

5. Consider two parallel infinite conducting rails on the $x - y$ plane, held a distance l apart and both extending along the x-axis. A metal rod of mass m is placed on the rails, with the ends of the rod on the rails. The rod is sliding towards the positive x-direction. There is a uniform magnetic field $\vec{B} = -B_0\hat{e}_z$, into the page. A resistance R is attached across the rails. See Figure 1.

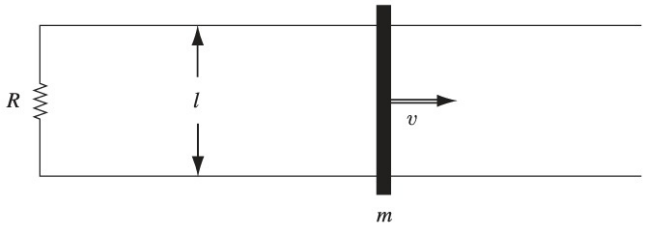

Figure 1: A metal rod sliding on two parallel conducting rails.

   (a) If the rod is moving at a given instant with velocity $\vec{v} = v\hat{e}_x$, find the instantaneous current across the resistor. In which direction does the current flow?

   (b) At the same instant, find the magnetic Lorentz force on the rod. In which direction is it?

   (c) Suppose the initial speed of the rod was v_0. Find its velocity at time $\tilde{t}$ in terms of v_0 and other parameters of the problem.

   (d) What is the total energy lost through Joule Heating in the resistor?

4 + 3 + 5 + 3 = 15 points